Timester Challenge
Harder Functions

Given $f(x) = x^2 - 3$ and $g(x) = 2x + 4$ find $(g \circ f)(x)$.

Bronze 

The functions f and g are such that $f(x) = 3x + 6$ and $g(x) = 7 - 2x$
Show that $fg(2) = 15$

Bronze 

For all values of x, $f(x) = x^2 + 3$
$g(x) = x - 2$.

a) Show that $fg(x) = x^2 - 4x + 7$

b) Solve $fg(x) = gf(x)$

Silver 

The functions f and g are such that $f(x) = 4x + 2$ and $g(x) = 2 - 4x$

Prove that $f^{-1}(x) + g^{-1}(x) = 0$

Gold 

Timester Challenge

Harder Functions

Answers

Given $f(x) = x^2 - 3$ and $g(x) = 2x + 4$ find $(g \circ f)(x)$.

$(g \circ f)(x) = g(f(x))$

$= g(x^2 - 3)$

$= 2(x^2 - 3) + 4$

$= 2x^2 - 6 + 4$

$= 2x^2 - 2$

Bronze

The functions f and g are such that $f(x) = 3x + 6$ and $g(x) = 7 - 2x$

Show that $fg(2) = 15$

$fg(x) = 3(7 - 2x) + 6$

$fg(x) = 21 - 6x + 6$

$fg(x) = 27 - 6x$

$fg(2) = 27 - 6 \times 2$

$fg(2) = 15$

Bronze

For all values of x, $f(x) = x^2 + 3$ $g(x) = x - 2$.

a) Show that $fg(x) = x^2 - 4x + 7$

$fg(x) = (x - 2)^2 + 3$

$= x^2 - 4x + 4 + 3$

$= x^2 - 4x + 7$

b) Solve $fg(x) = gf(x)$

$gf(x) = x^2 + 3 - 2$

$= x^2 + 1$

$\therefore \quad x^2 - 4x + 7 = x^2 + 1$

$-4x = 1 - 7$

$-4x = -6$

$4x = 6$

$x = \frac{6}{4} = 1\frac{1}{2}$

Silver

The functions f and g are such that $f(x) = 4x + 2$ and $g(x) = 2 - 4x$

Prove that $f^{-1}(x) + g^{-1}(x) = 0$

$$f^{-1}(x) = \frac{x - 2}{4}$$

$$g^{-1}(x) = \frac{x - 2}{-4} = \frac{2 - x}{4}$$

$$\frac{x - 2}{4} + \frac{2 - x}{4} = 0$$

$$\frac{x - x - 2 + 2}{4} = 0$$

Gold